A STATISTICAL METHOD FOR THE EMPLOYMENT OF FRAGMENT HIT AREA IN PENETRATION EQUATIONS OR CODES

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A statistical technique is introduced that yields a considerable enhancement of the prediction of damage to structures or military targets by means of fragments that emanate from warheads or shells.

1   INTRODUCTION

Arena trials with scaled warheads have been performed to support ballistic trials on frigates and vulnerability/effectiveness studies. These tests have been simulated with the PML-TNO vulnerability code. The poor simulation results demanded an analysis of the used techniques. After analysis a new statistical technique is introduced that delivers a different input for the penetration relations. This paper describes the experiment, the simulation, the analysis, the new technique and its results.

2   PROBLEM DEFINITION: EXPERIMENT AND SIMULATION

An arena trial was used to test the first modules of the PML-TNO TARget Vulnerability Assessment Code (TARVAC). This code is used in target vulnerability and weapon effectiveness studies. A scaled warhead, as depicted in Figure 1, was positioned in the centre of the test set-up. In this figure m indicates fragment mass, v the initial fragment velocity and K the shape number that will be defined later. The warhead was designed to accelerate preformed fragments of 1 gram to a velocity of about 1700 m/s.
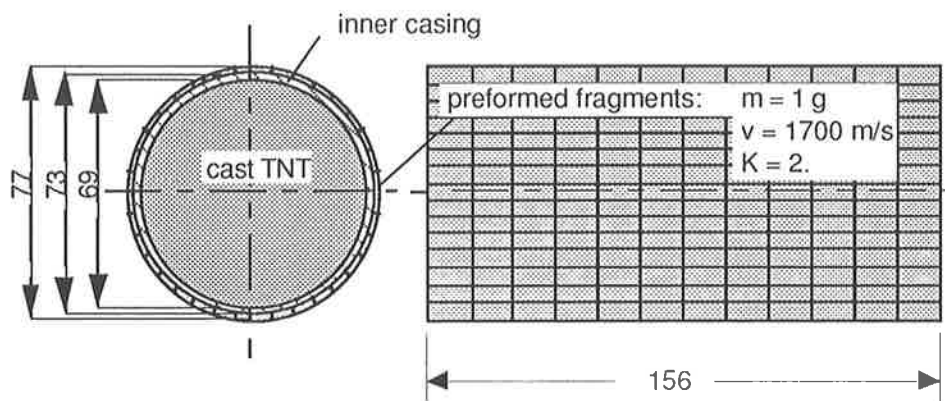

Figure 1   The test warhead (dimensions in mm)

The inner casing of the warhead, which had to prevent premature escape of the detonation gasses, produced natural fragments which are not addressed in this paper.

The preformed fragments had a rectangular shape which is representative for natural fragments. The choice for the ratios of the fragment dimensions are based upon studies performed at the Naval Surface Weapons Center [1].

The warhead was circumferenced by three steel plates with thicknesses of 4, 6 and 8 mm respectively, at a distance of 1 m from the warhead. The angular distribution and the velocities were determined by means of X-ray photography. The number of perforations was determined after the test, the results are given in Table 1.

The arena test was then simulated with TARVAC modules. Figure 2 shows a graphical presentation of a simulation. In Table 1 the results are compared to the experimental data and the poor agreement is apparent.
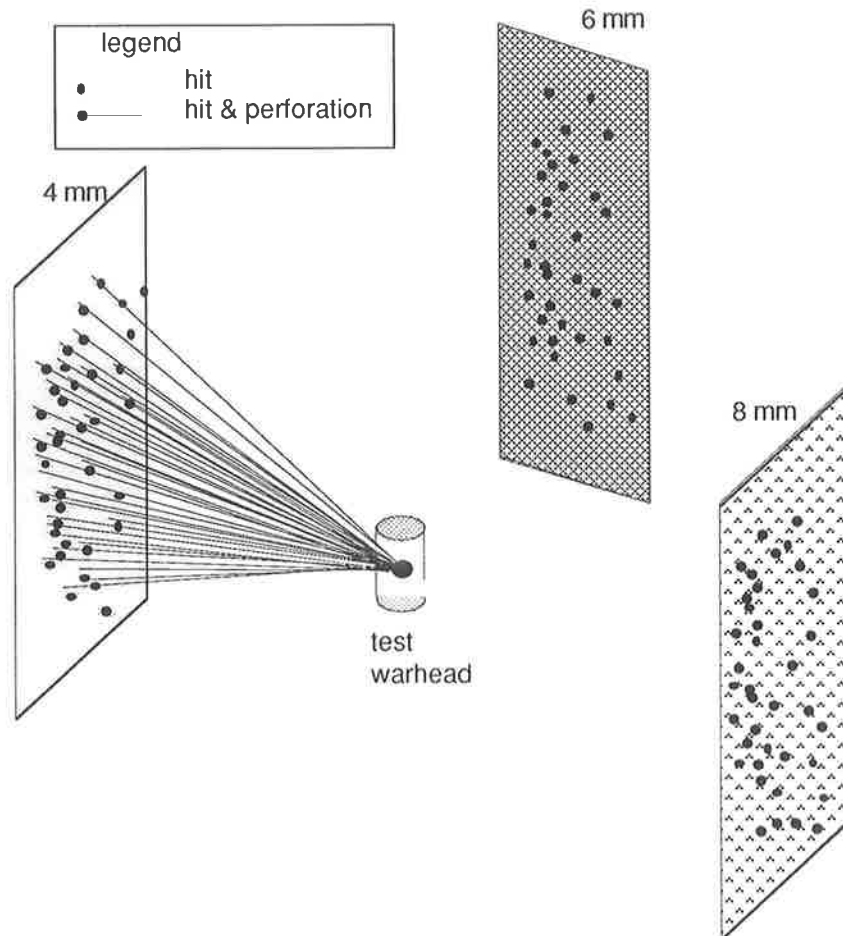

Figure 2   The graphical representation of the TARVAC simulation

Table 1   The number of perforations in experiment and simulation

T (mm)	experiment	simulation
4	40	37
6	17	0
8	8	0

The large discrepancy found indicated either a fault in the code or a too simple approach of the phenomena. Figure 3 gives the flowchart of the first modules of TARVAC. The input for the first four modules is quite accurately known and requires straightforward mathematics. So, a closer look at module no. 5 is appropriate, i.e. the use of the penetration relations.
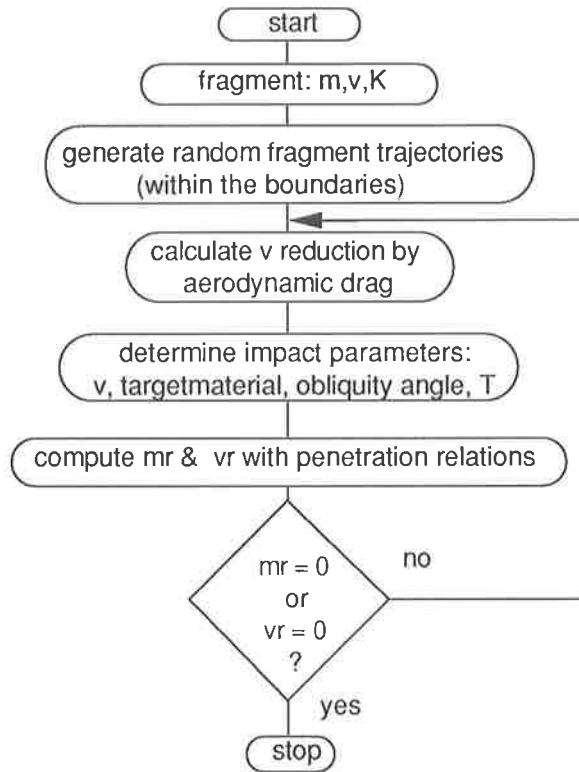

Figure 3   Flowchart of the first TARVAC modules.

In this particular TARVAC simulation the Thor equations [2] are used, but the same problems will arise with any other penetration relations. The Thor equations are:

$$v_r = v - 10^{c1}(TA)^{c2}m^{c3}\cos\theta^{-c4}v^{c5} \tag{1}$$

$$m_r = m - 10^{c6}(TA)^{c7}m^{c8}\cos\theta^{-c9}v^{c10} \tag{2}$$

According to these empirical equations the residual mass (m_r) and velocity (v_r) after perforation are dependent on:

- penetrator hit velocity (v) and mass (m)
- penetrator obliquity angle (θ)
- penetrator hit area (A)
- the target material; constants c_1 to c_{10}
- the target thickness (T)

All these input data are quite accurately known, except the penetrator hit area. Since it is not known at which orientation, i.e. with what presented cross-sectional area, the fragments will hit the target, it is common practice to employ the average presented area. The average presented area is determined by means of the non dimensional shape number (K).

In order to get more accurate results, in the number of perforation predictions, it is suggested to choose a random orientation of the fragment at impact and to calculate the presented area. To choose a random orientation the following technique has been used. A fragment is placed in the origin of a virtual sphere, as depicted in Figure 5 (For reasons of symmetry, only one-eighth of the sphere is displayed).
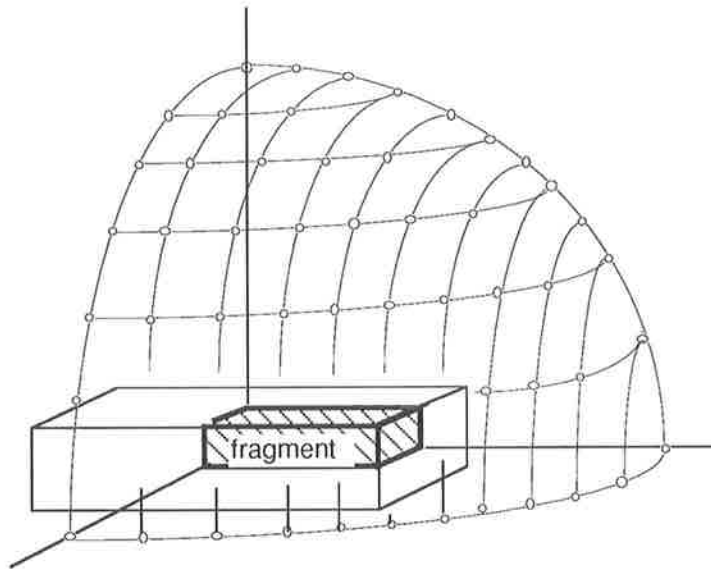

Figure 5   The equidistant distribution of observations

Over the circumference of this sphere an equidistant distribution of positions is placed. A random orientation can be obtained by choosing randomly a point from this equidistant distribution. The presented area for this observation point is then determined, see Figure 6.
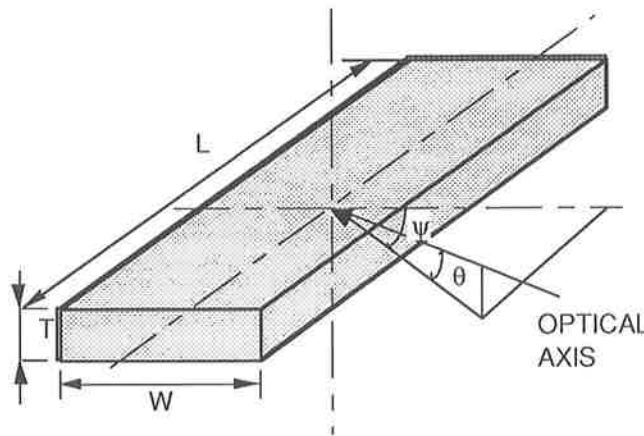

Figure 6   The presented fragment area

After calculation of the azimuth (ψ) and elevation (θ) angles of the direction of the observation, the presented area of a rectangular shaped fragment can be determined with :

$$\text{presented area} = WL\sin\theta + WT\cos\theta\sin\psi + LT\cos\theta\cos\psi, \qquad (5)$$

with L,W and T length, width and thickness respectively.

$$A_m = K \, V^{2/3} = K \left(\frac{m}{\rho} \right)^{2/3} \qquad [m^2] \qquad\qquad (3)$$

Here, V and ρ respectively are the volume and specific density of the fragment.

Often a shape factor (C_s) is used with the dimensions of ft^2 $grain^{1/3}$/lb, defined by the relation:

$A_m = C_s \, m^{2/3}$, instead of the non dimensional shape number K. In Annex A the shape number is calculated for rectangular shaped fragments.

Substitution of (3) in (1) and substitution of the material constants for steel yields for the penetration capacity in steel at normal impact (T_{50}):

$$T_{50} = Constant \, \frac{m^{0.4} \, v^{1.1}}{A} \qquad [m] \qquad\qquad (4)$$

Equation (4) and Figure 4 give a clear picture of the influence of the hit area on the penetration capacity. It is evident from Figure 4 and equation (4) that the influence of the hit area on the penetration capacity (T_{50}) is substantial.

It is also clear that when using the average presented area ($A/A_m = 1$) in the Thor equations, the fragments considered, will all perforate 4 mm steel plates, but will all be unable to perforate 6 mm steel. This is the reason for the poor TARVAC simulation results. The code does not take into account variations in presented area, so that all impacts (under equal m,v and θ) produce the same result. In reality, of course, there is considerable spread in presented areas and consequently also in perforation performance. In the next paragraph a statistical method will be introduced to take these variations into account.
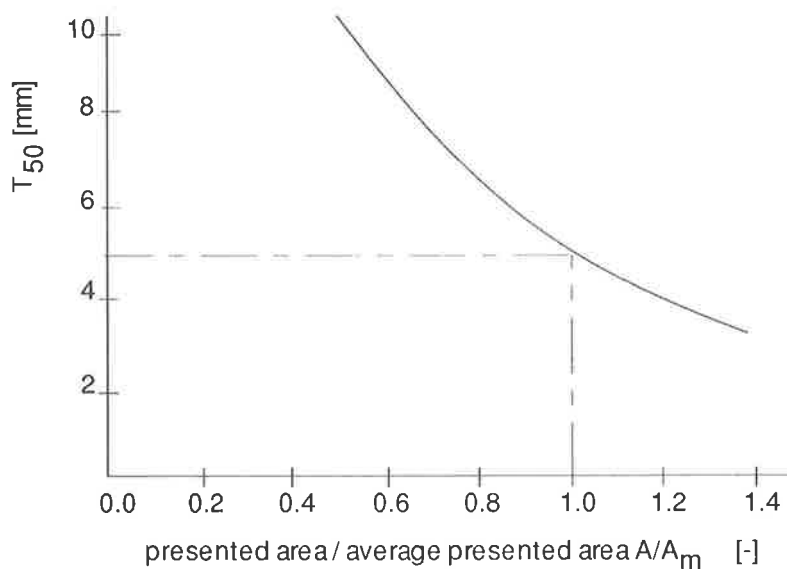

Figure 4   Influence of the presented area on the penetration capacity according to Thor.

From equation (3) the following specific values are calculated:

minimum area = WT   (6)

average area = 0.5 (LW + LT + WT)   (7)

$$\text{maximum area} = \sqrt{(LW)^2 + (TL)^2 + (TW)^2} \quad\quad (8)$$

variance =

$$\frac{1}{12}[(LT)^2 + (WT)^2 + (LW)^2] + [\frac{4}{3\pi} - \frac{1}{2}][LW(T)^2 + TW(L)^2 + TL(W)^2] \quad\quad (9)$$

$$\text{standard deviation} = \sqrt{\text{variance}} \quad\quad (10)$$

If an observation is made in every point a frequency distribution of the possible presented areas can be obtained. Figure 7 shows the fragment area frequency distribution of the preformed fragment here considered.
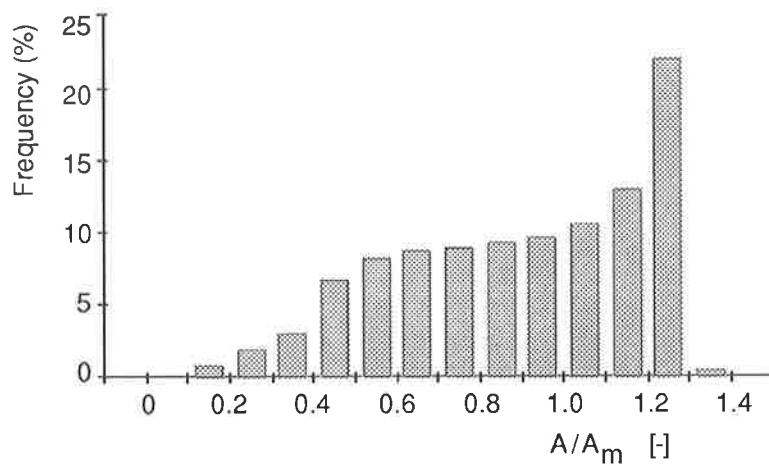

Figure 7   The frequency distribution of the ratio A/A_m for the fragment K = 2.

It is interesting to notice that for this preformed fragment, representative for a natural fragment, the minimum area is only 20 % of the average while the maximum is 40 % larger than the average. Besides the mode class (class with largest frequency) is 30 % larger than the average value. According to Figure 4 this will give a penetration capacity ranging from 3 to more than 8 mm steel.

The technique described has been added to the TARVAC code according to the flowchart of Figure 8.
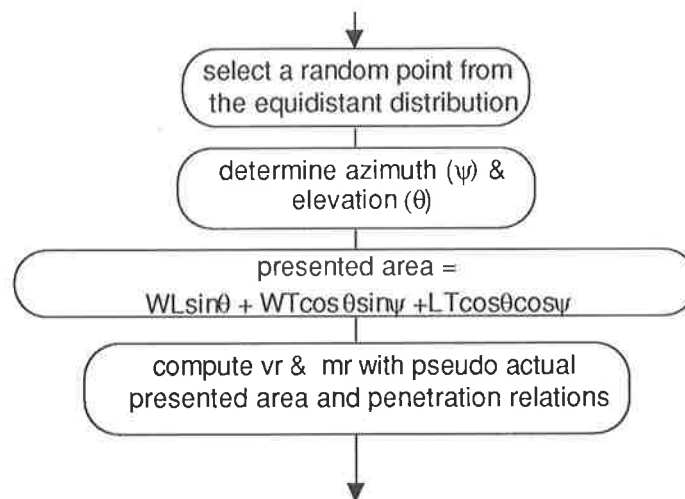

Figure 8   The addition to the TARVAC flowchart.

The randomly chosen impact area can be used as input for the Thor equations (or in any other equation or penetration code).

5 RESULTS

After the new technique has been implemented in TARVAC a re-run of the simulations has been performed. Figure 9 and Table 2 show the remarkable improvement of the results of the new simulations.

Table 2   The number of perforations in experiment and simulation after implementation of the new technique

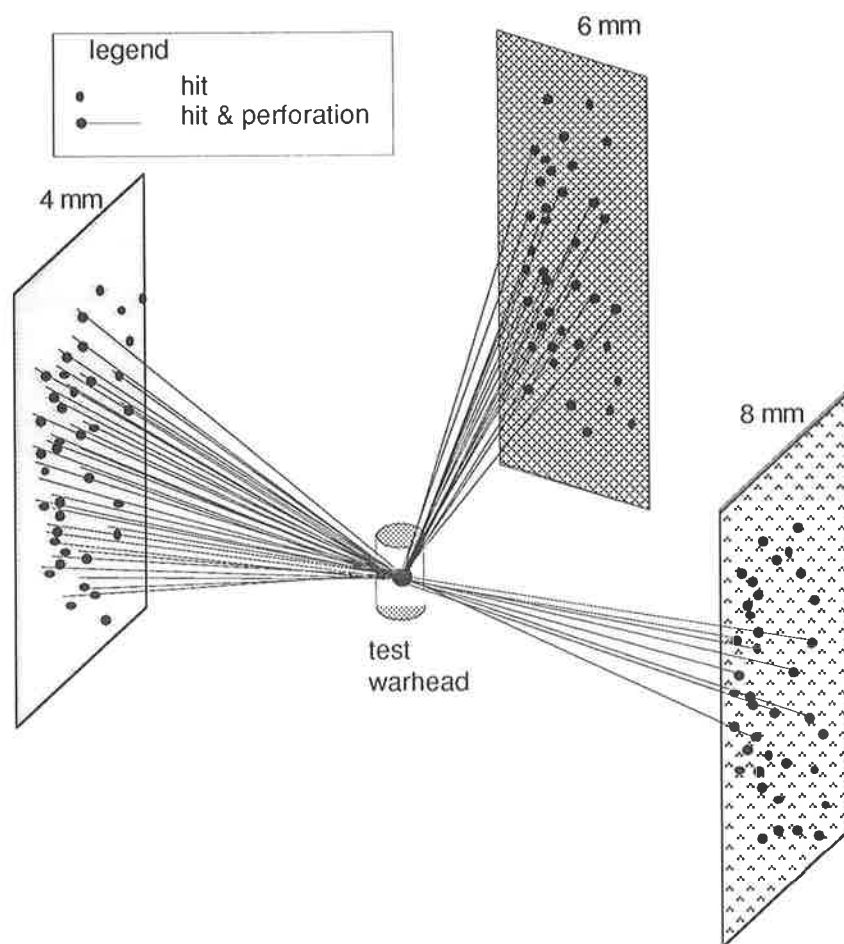| T (mm)	experiment	simulation
4	40	37
6	17	16
8	8	8

Figure 9   The graphical representation of the improved TARVAC simulation.

6 CONCLUSION

In most vulnerability assessment codes the average presented area is used in the penetration relations. This can lead to incorrect results. A statistical method has been introduced that takes the random orientation of fragments into account. A more accurate representation of reality is obtained leading to a better prediction of the number of perforations and thus to more reliable vulnerability studies.

REFERENCES

[1] McCleskey, F.
      Drag coefficients for irregular fragments.
      Naval Surface Weapons Center, Dahlgren, Virginia, U.S.A.

[2] Project Thor
      The resistance of various metallic materials to perforation by steel fragments;
      empirical relationships for fragment residual velocity and residual weight.
      Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, U.S.A.
      technical report No. 47, April 1961

ANNEX A: THE DETERMINATION OF THE SHAPE NUMBER FOR A
            RECTANGULAR SHAPED FRAGMENT.

The shape number (K) of a fragment is defined by:

$$K = \frac{A_m}{V^{2/3}} \qquad [\text{-}] \qquad (A1)$$

where:

A_m = average presented area   [m^2]
V = volume of the fragment   [m^3]

The average presented area can be calculated by means of the rule of Cauchy: The average presented area of a convex body equals one-fourth of the total area.

For a rectangular shaped fragment with dimensions L, W and T, this is:

$$A_m = \frac{2TW + 2LT + 2LW}{4} \qquad . \qquad (A2)$$

Substitution of (A2) in (A1) gives:

$$K = \frac{TW + LT + LW}{2(LWT)^{2/3}} \qquad . \qquad (A3)$$

Defining $\alpha = \frac{L}{T}$ and $\beta = \frac{W}{T}$ changes (A3) in:

$$K = \frac{1}{2}\left[\frac{\beta^{1/3}}{\alpha^{2/3}} + \frac{\alpha^{1/3}}{\beta^{2/3}} + \alpha^{1/3}\beta^{1/3} \right] \qquad . \qquad (A4)$$

Rearranging then yields:

$$K = \frac{1}{2}(\alpha\beta)^{1/3}\left[1 + \frac{1}{\alpha} + \frac{1}{\beta} \right] \qquad .$$